What You Will Learn

- Find angle measures in regular polygons.
- Find areas of regular polygons.

\[ A = \frac{bh}{2} = \frac{1}{2} bh \]

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Finding Angle Measures in Regular Polygons

The diagram shows a regular polygon inscribed in a circle.

The **center of a regular polygon** and the **radius of a regular polygon** are the center and the radius of its circumscribed circle.

The distance from the center to any side of a regular polygon is called the **apothem of a regular polygon**. The apothem is the height to the base of an isosceles triangle that has two radii as legs. The word “apothem” refers to a segment as well as a length. For a given regular polygon, think of *an* apothem as a segment and *the* apothem as a length.

A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices of the polygon. To find the measure of each central angle, divide 360° by the number of sides.
In the diagram, polygon $ABCD$ is a regular decagon inscribed in $\odot P$. Find each angle measure.

a. $m\angle KJP = 36^\circ$

b. $m\angle LPK = 18^\circ$

c. $m\angle LJP = 72^\circ$

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In the diagram, $WXYZ$ is a square inscribed in $\odot P$.

3. Identify the center, a radius, an apothem, and a central angle of the polygon.

4. Find $m\angle XPY$, $m\angle XPQ$, and $m\angle PXQ$.

- Center: $P$
- Radius: $\overline{XP}$ or $\overline{YP}$
- Apothem: $\overline{PQ}$
- Central $\angle$: $\angle XPY$

$\angle XYP$:
- $\frac{360}{4} = 90^\circ$
- $90 + x + 45 = 180$
- $x = 45^\circ$

$\angle XPQ$:
- $90^\circ$

$\angle PXQ$:
- $45^\circ$

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$A = \frac{1}{2}bh$  
SOH-CAH-TOA
Finding Areas of Regular Polygons

You can find the area of any regular \( n \)-gon by dividing it into congruent triangles.

\[
A = \text{Area of one triangle} \times \text{Number of triangles}
\]

\[
= \left( \frac{1}{2} \cdot s \cdot a \right) \times n
\]

Base of triangle is \( s \) and height of triangle is \( a \). Number of triangles is \( n \).

\[
= \frac{1}{2} \cdot a \cdot (n \cdot s)
\]

Commutative and Associative Properties of Multiplication

\[
= \frac{1}{2} a \cdot P
\]

There are \( n \) congruent sides of length \( s \), so perimeter \( P \) is \( n \cdot s \).

Area of a Regular Polygon

The area of a regular \( n \)-gon with side length \( s \) is one-half the product of the apothem \( a \) and the perimeter \( P \).

\[
A = \frac{1}{2} aP, \text{ or } A = \frac{1}{2} a \cdot ns
\]
A regular hexagon is inscribed in a circle with a diameter of 32 units. Find the area of the hexagon.

\[ A = \frac{1}{2} aP \]
\[ = \frac{1}{2} 8\sqrt{3} 96 \]
\[ = 4\sqrt{3} 96 \]
\[ A = 384\sqrt{3} \]

\[ a = 8\sqrt{3} \]
\[ P = 96 \]

A mirror is in the shape of a regular nonagon with 6-inch sides. What is the area of the mirror?

\[ \text{SOH-CAH-TOA} \]
\[ \cos \theta \quad 20 = \frac{3}{a} \]
\[ a = 8.2 \]
\[ \sin \theta = \frac{3640}{a} \quad 3640 a = 3 \quad 3640 \]
\[ a = 8.2 \]
\[ A = \frac{1}{2} aP \]
\[ = \frac{1}{2} 8.2 \cdot 54 \]
\[ = 221.4 \cdot 1 \]
\[ A = 221.4 \text{ in}^2 \]